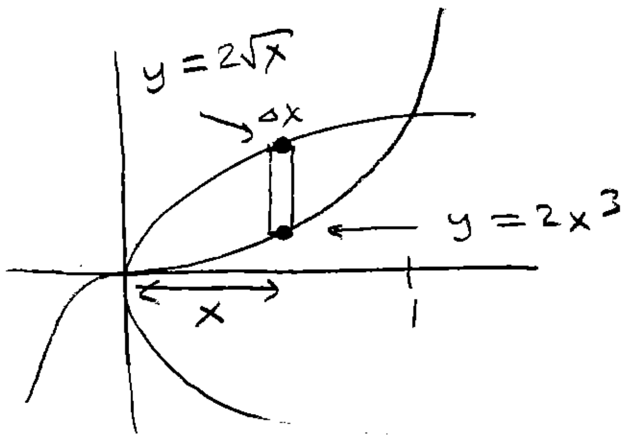
Closing Wed: HW_3A,3B,3C (6.1-6.3)
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:
Find the area of the region bounded by $4x = y^2$ and $y = 2x^3$ in 2 ways:
(i) Using dx
(ii) Using dy



$$\int_0^1 2\sqrt{x} - 2x^3\,dx$$

$$\frac{4}{3}x^{3/2} - \frac{2}{4}x^4 \Big|_0^1$$

$$4/3 - \frac{1}{2} = \frac{8}{6} - 3/6 = \boxed{5/6}$$

$$4x = y^2 \iff \begin{cases} y = -2\sqrt{x} \\ y = 2\sqrt{x} \end{cases}$$

$$x = \left(\tfrac{1}{2}y\right)^{1/3} \iff y = 2x^3$$

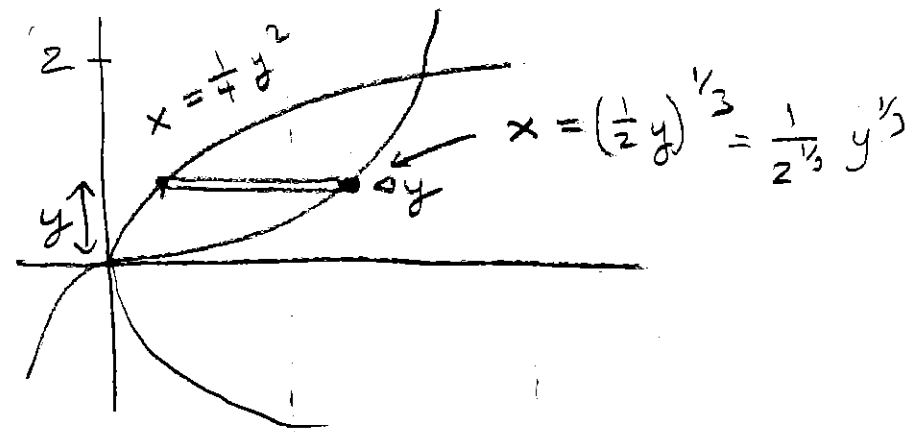
INTERSECTIONS $\Rightarrow$ $4x \stackrel{?}{=} (2x^3)^2$

$$4x = 4x^6$$

$$x = x^6$$

$$0 = x^6 - x = x(x^5 - 1)$$

$x = 0$ or $x = 1$



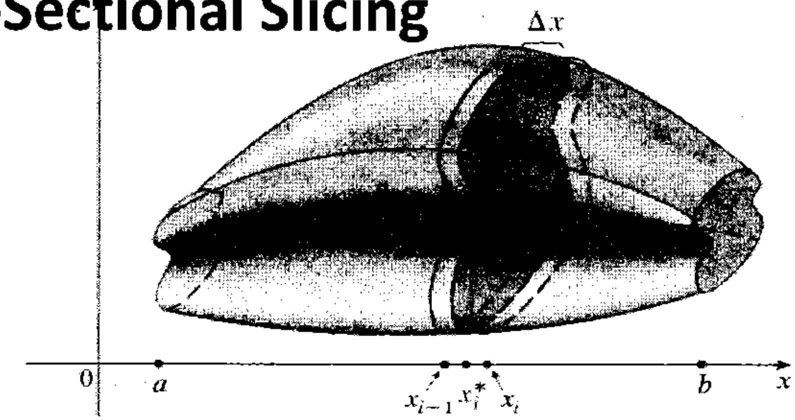
$$\int_0^2 \left(\tfrac{1}{2}\right)^{1/3} y^{1/3} - \frac{1}{4}y^2\,dy$$

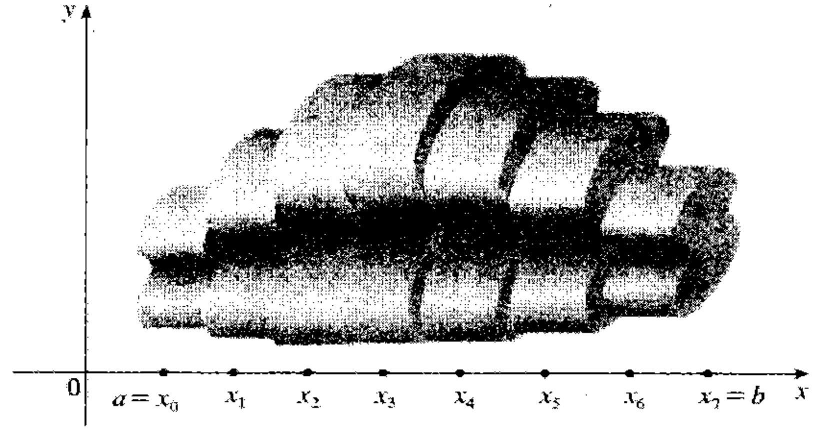
$$= \left(\tfrac{1}{2}\right)^{1/3}\frac{3}{4}y^{4/3} - \frac{1}{12}y^3 \Big|_0^2$$

$$= \left(\tfrac{1}{2}\right)^{1/3}\frac{3}{4}(2)^{4/3} - \frac{1}{12}(2)^3$$

$$= \frac{6}{4} - \frac{8}{12} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \boxed{5/6}$$

6.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice } \approx \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

$$\text{Total Volume } \approx \sum_{i=1}^{n} \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

We conclude

$$\text{Volume } = \int_a^b A(x)dx =$$

$$\int_a^b \text{"Cross-sectional area formula"}dx$$

Volume using cross-sectional slicing

1. Draw region. Cut **perpendicular** to rotation axis. Label *x* if that cut crosses the *x*-axis (and *y* if *y*-axis). Label **everything** in terms this variable.

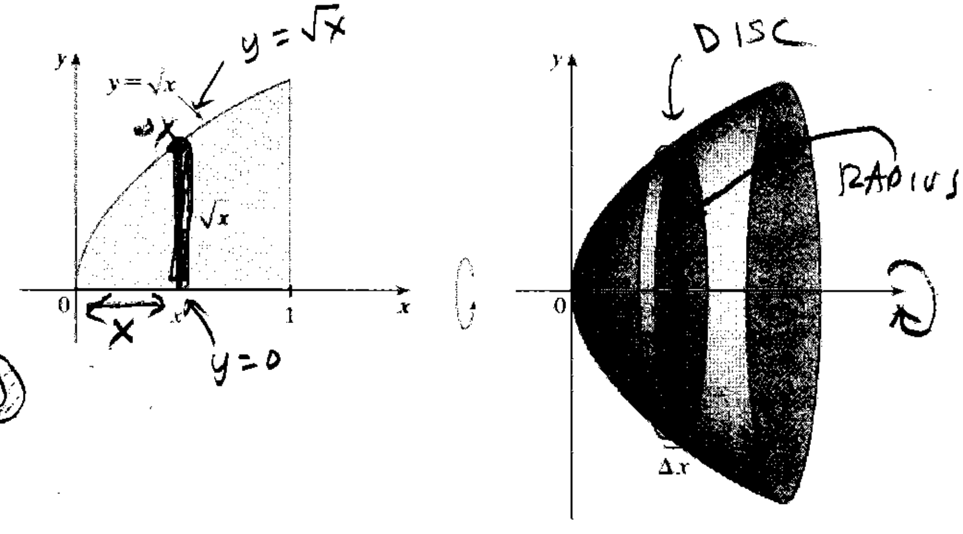
2. Formula for cross-sectional area?

disc:	Area = π(radius)^2
washer:	Area = π(outer)^2 - π(inner)^2
square:	Area = (Height)(Length)
triangle:	Area = ½ (Height)(Length)

SAME

3. Integrate the area formula.

Example: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the **x-axis**.



(a) (b)

$$\int_0^1 \pi\,(\text{RADIUS})^2\,dx$$

$$= \int_0^1 \pi\,(\sqrt{x})^2\,dx$$

$$= \pi \int_0^1 x\,dx = \pi\,\tfrac{1}{2}x^2\Big|_0^1$$

$$= \boxed{\pi/2}$$

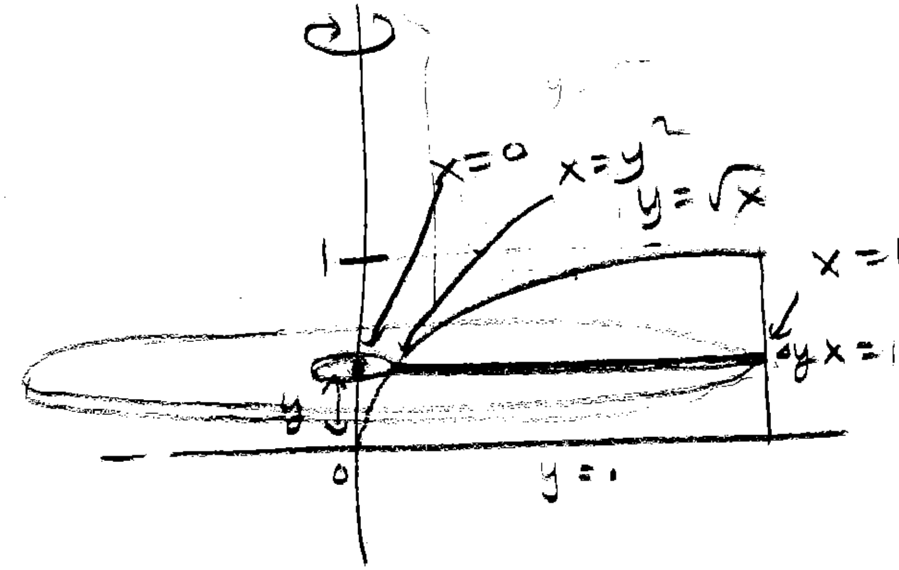
Example: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the ***y*-axis**.

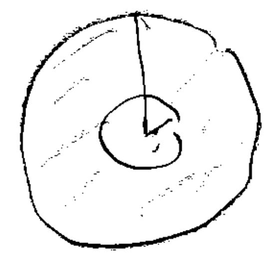
$$\int_0^1 \pi(1)^2 - \pi(y^2)^2\,dy$$

$$= \pi\int_0^1 1 - y^4\,dy$$

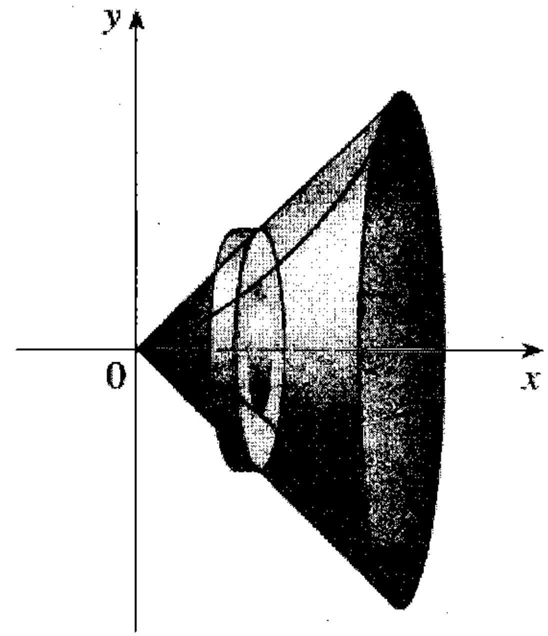
$$= \pi\left(y - \frac{1}{5}y^5\Big|_0^1\right)$$

$$= \pi(1 - 1/5) = \boxed{4\pi/5}$$





Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. Find the volume of the solid obtained by rotating R about the **x-axis**.

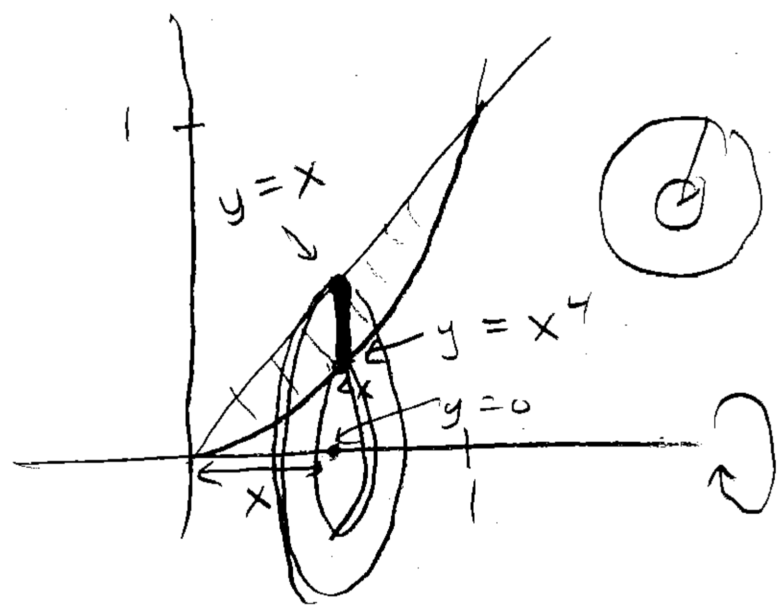


$$\int_0^1 \pi(x)^2 - \pi(x^4)^2\,dx$$

$$\pi\int_0^1 x^2 - x^8\,dx$$

$$\pi\left(\frac{1}{3}x^3 - \frac{1}{9}x^9\right)\Big|_0^1$$

$$\pi\left(\frac{1}{3} - \frac{1}{9}\right) = \boxed{\frac{2\pi}{9}}$$

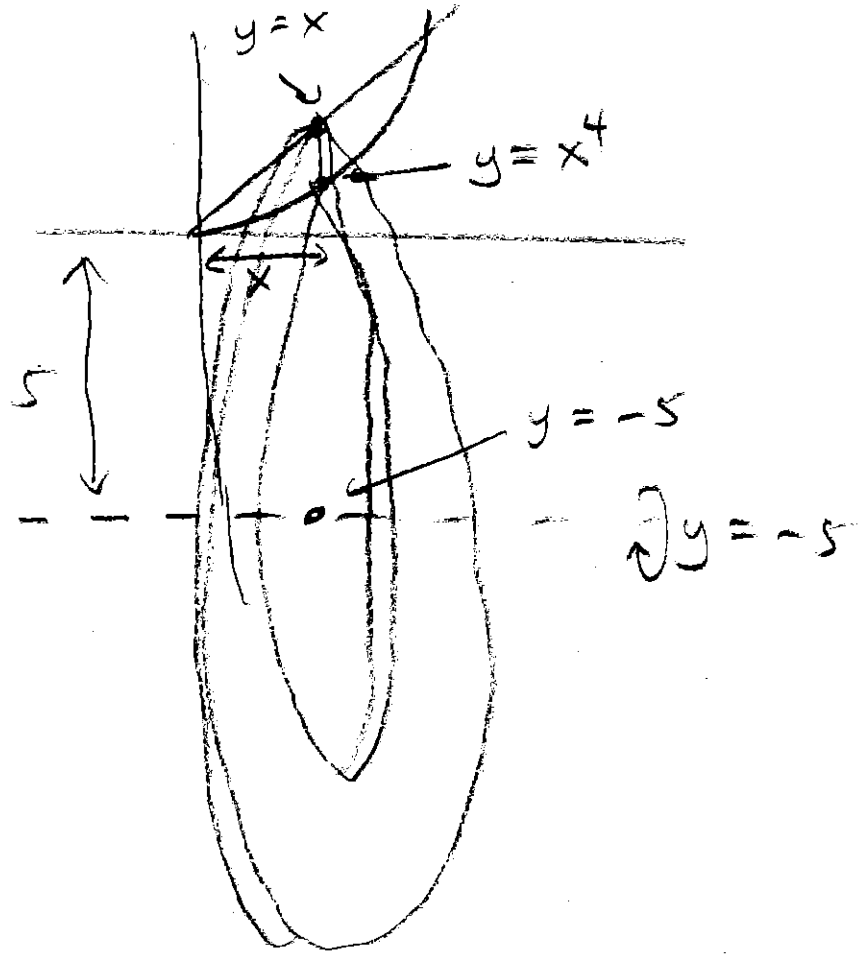


Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. (R is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

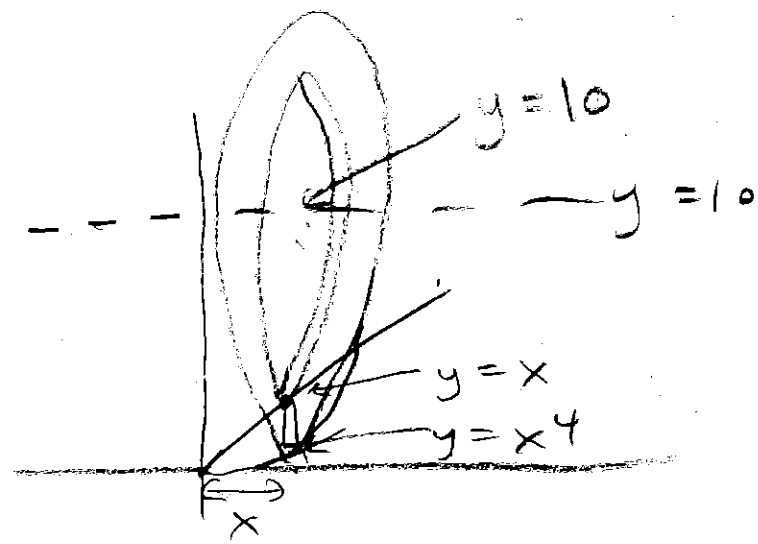
$$\int_0^1 \pi (x - -5)^2 - \pi (x^4 - -5)^2 \, dx$$

$$\pi \int_0^1 (x+5)^2 - (x^4+5)^2 \, dx$$



(b) Now rotate about the horizontal line y = 10. What changes?

$$\int_0^1 \pi (10 - x^4)^2 - \pi (10 - x)^2 \, dx$$

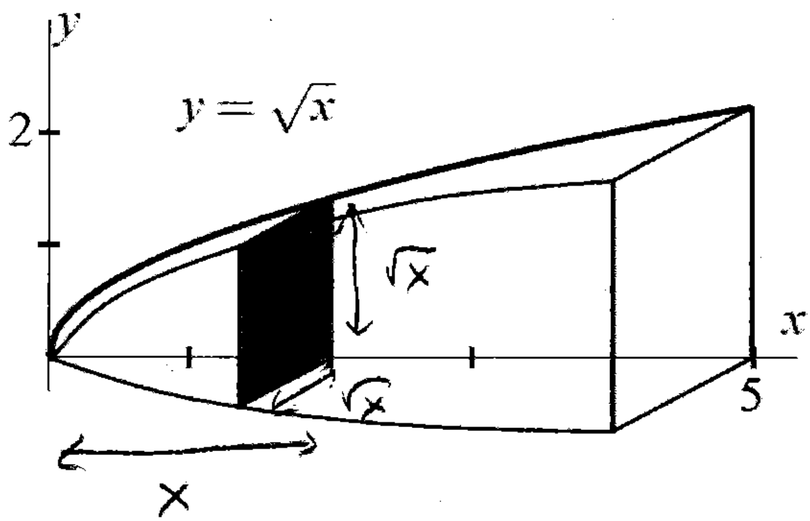


Example:

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



$$\int_0^5 (\text{HEIGHT})(\text{LENGTH})\,dx$$

$$\int_0^5 \sqrt{x}\;\sqrt{x}\;dx$$

$$\int_0^5 x\,dx = \frac{1}{2}x^2\Big|_0^5 = \frac{1}{2}25 = \boxed{\frac{25}{2}}$$

;ummary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has a major limitation:

;.2 method about *x-axis*, must use *dx*.

;.2 method about *y-axis*, must use *dy*.

Nhat if the regions is rotated about
:he *x*-axis and we need to use *dy*?
or about *y*-axis and we need dx?)

n these cases, 6.2 "Cross-sectional
;licing" wouldn't work!

Ne need another method.
'hat is what we will do in 6.3.

Close Wed: HW_3A,3B,3C

(complete sooner!)

Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:

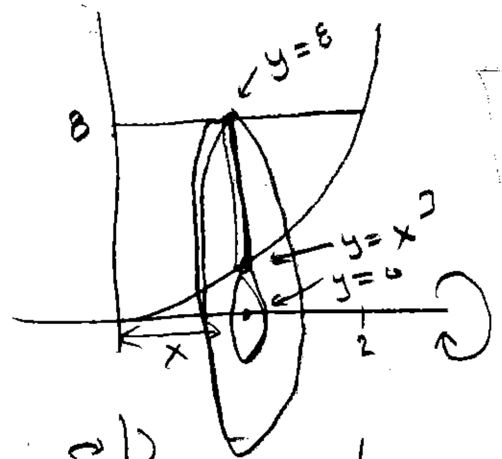
Consider the region *R* bounded by $y = x^3, y = 8$, and $x = 0$.

Set up the integrals that would give the volume of the solid obtained by rotating *R* about the ....

(a) ... *x*-axis.

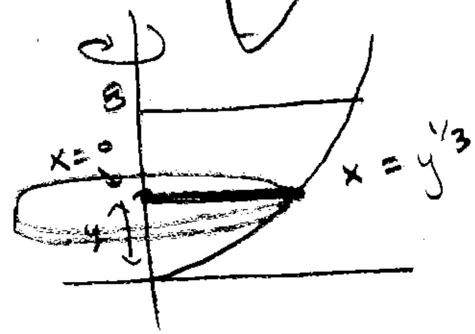
(b) ... *y*-axis.

(c) ... vertical line *x* = -10.



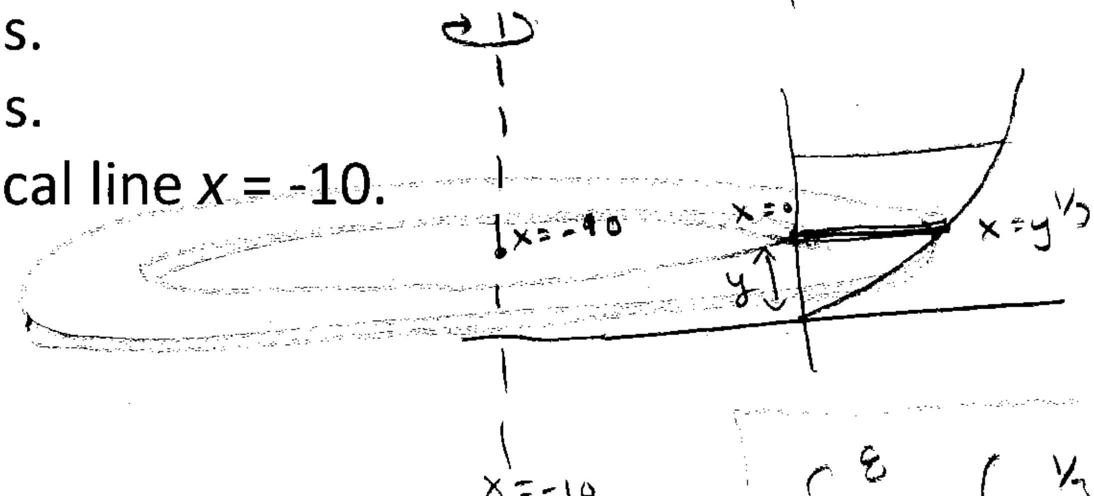
$$\int_0^2 \pi(8)^2 - \pi(x^3)^2\,dx$$

$$\pi\int_0^2 64 - x^6\,dx$$



$$\int_0^8 \pi\left(y^{1/3}\right)^2 dy$$

$$= \pi\int_0^8 y^{2/3}\,dy$$



$$\int_0^8 \pi\left(y^{1/3}+10\right)^2 - \pi(10)^2\,dy$$